Chapter 2: LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to
- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

More help on solving equations can be obtained by downloading the leaflet available at this website: http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf

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Example 1: Solve the equation $\quad 64 - 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the x term is positive): $\qquad 64 = 3x + 25$

Step 2: Subtract 25 from both sides: $\qquad 39 = 3x$

Step 3: Divide both sides by 3: $\qquad 13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:
Step 1: Begin by adding $2x$ to both sides $\qquad 8x + 7 = 5$
(to ensure that the x terms are together on the same side)

Step 2: Subtract 7 from each side: $\qquad 8x = -2$

Step 3: Divide each side by 8: $\qquad x = -\frac{1}{4}$

Exercise A: Solve the following equations, showing each step in your working:

1) $\quad 2x + 5 = 19$

2) $5x - 2 = 13$

3) $11 - 4x = 5$

4) $\quad 5 - 7x = -9$

5) $11 + 3x = 8 - 2x$

6) $7x + 2 = 4x - 5$

Example 3: Solve the equation $\quad 2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $\qquad 6x - 4 = 20 - 3x - 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $\qquad 6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $\qquad 9x - 4 = 14$

Step 4: Add 4: $\qquad 9x = 18$

Step 5: Divide by 9: $\qquad x = 2$

Exercise B: Solve the following equations.

1) $\quad 5(2x - 4) = 4$

2) $\quad 4(2 - x) = 3(x - 9)$

3) $\quad 8 - (x + 3) = 4$

4) $\quad 14 - 3(2x + 3) = 2$

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\dfrac{y}{2} + 5 = 11$

Solution:
Step 1: Multiply through by 2 (the denominator in the fraction): $\quad y + 10 = 22$

Step 2: Subtract 10: $\qquad\qquad\qquad\qquad\qquad\qquad\qquad y = 12$

Example 5: Solve the equation $\dfrac{1}{3}(2x + 1) = 5$

Solution:
Step 1: Multiply by 3 (to remove the fraction) $\qquad 2x + 1 = 15$

Step 2: Subtract 1 from each side $\qquad\qquad\qquad 2x = 14$

Step 3: Divide by 2 $\qquad\qquad\qquad\qquad\qquad x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

Example 6: Solve the equation $\dfrac{x+1}{4} + \dfrac{x+2}{5} = 2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator

$$\dfrac{20(x+1)}{4} + \dfrac{20(x+2)}{5} = 40$$

Step 3: Simplify the left hand side:

$$\dfrac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \dfrac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$$

$$5(x+1) + 4(x+2) = 40$$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 7: Solve the equation $x + \dfrac{x-2}{4} = 2 - \dfrac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x + \dfrac{12(x-2)}{4} = 24 - \dfrac{12(3-5x)}{6}$$

Simplify $12x + 3(x-2) = 24 - 2(3-5x)$

Expand brackets $12x + 3x - 6 = 24 - 6 + 10x$

Simplify $15x - 6 = 18 + 10x$

Subtract $10x$ $5x - 6 = 18$

Add 6 $5x = 24$

Divide by 5 $x = 4.8$

Exercise C: Solve these equations

1) $\dfrac{1}{2}(x+3) = 5$

2) $\dfrac{2x}{3} - 1 = \dfrac{x}{3} + 4$

3) $\dfrac{y}{4} + 3 = 5 - \dfrac{y}{3}$

4) $\dfrac{x-2}{7} = 2 + \dfrac{3-x}{14}$

Exercise C (continued)

5) $\dfrac{7x-1}{2} = 13 - x$

6) $\dfrac{y-1}{2} + \dfrac{y+1}{3} = \dfrac{2y+5}{6}$

7) $2x + \dfrac{x-1}{2} = \dfrac{5x+3}{3}$

8) $2 - \dfrac{5}{x} = \dfrac{10}{x} - 1$

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.
Therefore
$$n + (n + 1) + (n + 2) = 96$$
$$3n + 3 = 96$$
$$3n = 93$$
$$n = 31$$
So the numbers are 31, 32 and 33.

Exercise D:

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.